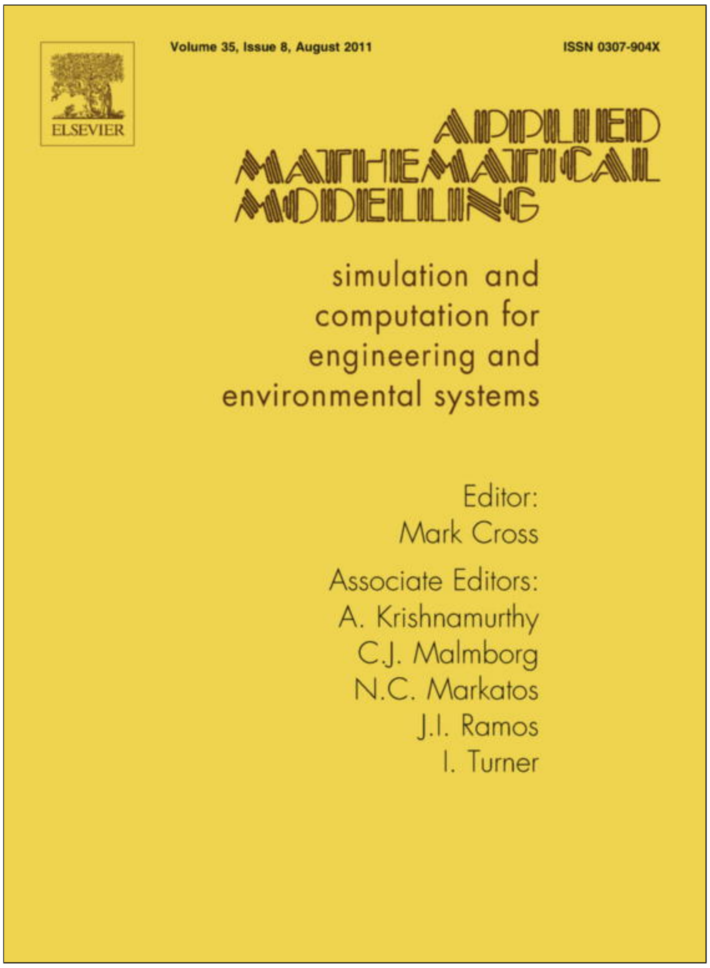
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# LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity

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ABSTRACT

In the present paper, an estimation is made to investigate the transient phenomena in magneto-thermoelastic model in the context of the theory of generalized thermoelasticity LS model with variable thermal conductivity. FEM is proposed to analyze the problem and obtain the numerical solutions for the displacement, temperature, and radial and hoop stresses. The boundary conditions for the mechanical and Maxwell's stresses at the internal and outer surfaces is considered. An application of an infinitely long annular cylinder is investigated for the inner surface is traction free and subjected to thermal shock, while the outer surface is traction free and thermally isolated. Finally, the displacement, incremental temperature, the stress components are obtained and then presented graphically.


## 1. Introduction

Recently, more attentions has been made for the theory of thermoelasticity because of its utilitarian aspects in diverse fields, especially, engineering, structures, geology, biology, geophysics, acoustics, physics, plasma, etc. Duhamel [1] and Neumann [2] introduced the theory of uncoupled thermoelasticity which contain two shortcomings. First, the fact that the mechanical state of the elastic body has no effect on the temperature, is not in accordance with true physical experiments. Second, the heat equation being parabolic predicts an infinite speed of propagation for the temperature, which is not physically admissible. The theory of elasticity with nonuniform heat which was in half-space subjected of thermal shock in this context which known as the theory of uncoupled thermoelasticity and the temperature is governed by a parabolic partial differential equation in temperature term only has been discussed [3]. Biot [4] introduced the theory of classical thermoelasticity, the equation of motion is hyperbolic in nature, whereas the heat conduction equation is parabolic in nature; the theory predicts a finite speed for predominantly elastic disturbances but an infinite speed for predominantly thermal disturbances, which are coupled together. Obviously, this result is physically unrealistic, so, [5–11] made an experimental investigations conducted on various solids, for example, have shown that heat pulses do propagate with finite speed. These theories remove

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**Nomenclature**

| | |
|---|---|
| $\alpha_t$ | internal thermal expansion coefficient, $\gamma = \alpha_t(3\lambda + 2\mu)$ |
| $\theta$ | temperature increment, $\theta = \lvert T - T_o \rvert \ll 1$ |
| $\lambda$ and $\mu$ | Lamés constants |
| $\mu_e$ | magnetic permeability |
| $\sigma_{ij}$ | components of stress tensor |
| $\rho$ | density |
| $\tau_o$ | relaxation time |
| $\vartheta$ | mapping of $\theta$ |
| $C_e$ | specific heat per unit mass |
| $e_{ij}$ | components of strain tensor |
| $\underset{\to}{H}_o$ | constant magnetic field |
| $\underset{\to}{H}$ | magnetic field vector |
| $J$ | electric current density |
| $k$ | diffusivity |
| $K$ | thermal conductivity |
| $t$ | time |
| $T_o$ | reference temperature |
| $u_i$ | components of displacement vector |

the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity introduced by Biot [4]. Lord and Shulman [12] have discovered the theory which determines the finite speed for the motion due to thermal field using one relaxation time. By including temperature rate, Green and Lindsay [13] violated the classical Fourier's law of heat conduction when the body under consideration has a center of symmetry. This theory also predicts a finite speed of heat propagation using two relaxation times. This implies that the thermal wave propagates with infinite speed, a physically impossible result.

During the second half of 20th century, nonisothermal problems of the theory of elasticity became increasingly impact. This is due mainly to their many applications in widely diverse fields. First, in the nuclear field, the external high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations. Secondly, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure [14]. Nowacki [15] investigated the dynamic problems of thermoelasticity. Some problems of thermoelasticity are discussed [16,17]. Three different models of thermoelasticity in an alternative way have been discussed including the anisotropic case [18]. A survey article of representative theories in the range of generalized thermoelasticity is due to Hetnarski and Ignaczak [19].

Eraby and Suhubi [20] studied wave propagation in a cylinder. Ignaczak [21] studied a strong discontinuity wave and obtained a decomposition theorem [22]. Ezzat [23] has also obtained the fundamental solution for this theory. Many problems have been solved in the context of the generalized thermoelasticity by El-Maghraby and Yousef [24] and [25], Youssef et al. [26] and Yousef [27–29]. Modern structural elements are often subjected to temperature changes of such magnitude that their material properties may no longer be regarded as having constant values even in an approximate sense. The thermal and mechanical properties of materials vary with temperature, so that the temperature dependence of material properties must be taken into consideration in the thermal stress analysis of these elements [30–32].

In recent years, the theory of magneto-thermoelasticity which deals the interactions among strain, temperature and electromagnetic fields has drawn the attention of many researchers because of its extensive uses in diverse fields, such as geophysics for understanding the effect of the Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, development of a highly sensitive superconducting magnetometer, electrical power engineering, optics, etc. Knopoff [34] and Chadwick [35] studied these types of problems in the beginning and developed by Kaliski and Petykiewicz [36]. The generalized magneto-thermoelasticity in a perfectly conducting medium is investigated [37]. Baksi et al. [38] illustrate magneto-thermoelastic problems with thermal relaxation and heat sources in a three dimensional in.nite rotating elastic medium. In Yousef and Abbas [39], the in.uence of variable thermal conductivity, thermal shock and relaxation time for an annular cylinder has been discussed. Abd-Ahha et al. [40] and Abo-Dahab and Mohamed [41] illustrated some problems in magneto-thermoelasticity and viscosity.

The present paper is devoted to estimate the influence of relaxation time, magnetic field, thermal shock and variable thermal conductivity. LS theory of generalized thermoelasticity is considered under variable thermal conductivity and magnetic field. We consider an infinitely long annular cylinder whose inner surface is traction free and subjected to thermal shock and magnetic field. The outer surface is also traction free and thermally isolated. The medium parameters quiescent initial state. The FEM is proposed to obtain the displacement, temperature and the radial and hoop stresses. Finally, the results obtained are represented graphically.